Topological Gravity, the Hierarchy Problem and Axion Physics

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In the last years higher dimensional physics has won importance. Despite the Superstrings, higher dimensional effects, in measurable scales of energy (some TeV), became only possible with Randall-Sundrum's models (RS). In particular, recent studies in neutrino and axion physics have proposed new and interesting questions about neutrino mixings and new scales intermediating the Weak and Planck scales. In this work we discuss field theoretic models that in some aspects are similar to the RS models. Indeed, our models contain domain walls, solitonic-like objects that mimics the branes of the RS models. Applications are discussed ranging from topological field theories in higher dimensions until models containing $D = 5$ space-time torsion in the RS scenario. In particular, we talk about subjects related to topological gravity, the hierarchy problem and axion physics. The topological terms studied are generalizations for $D > 4$ of the axion-foton coupling in $D = 4$. Such procedure involves naturally the Kalb-Ramond field. By dimensional reductions we obtain topological terms of the $B \wedge F$ type in $D = 4$ Chern-Simons-like and $B \wedge \partial\varphi$ type both in $D = 3$.

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I. INTRODUCTION

A. The hierarchy problem

May the Standard Model be placed in form of the recent insights coming from String Theories, where several dimensions appear so naturally? The standard model for strong, weak and electromagnetic interactions, described by the gauge group $SU(3) \times SU(2) \times U(1)$, has its success strongly based on experimental evidences. However, it has several serious theoretical drawbacks suggesting the existence of new and unexpected physical facts beyond those discussed in the last years. One of these problems is the so called *gauge hierarchy problem* which is related to the weak (M_{ew}) and Planck (M_{pl}) scales, the fundamental scales of the model. The central idea of this problem is to explain the smallness of the hierarchy $M_{ew}/M_{pl} \sim 10^{-17}$. In the context of the minimal standard model, this hierarchy of scales is unnatural since it requires a fine-tuning order by order in the perturbation theory. The first attempts to solve this problem were the technicolor scenario[1] and the low energy supersymmetry[2]. We mention that electroweak interactions have been proved at distances M_{ew}^{-1}, but gravity has only accurately measured in the $1cm$ range. Note that the Planck length is $10^{-33}cm$.

B. Extra dimensions - Randall-Sundrum scenario

With the string theories, the search of many-dimensional theories became important. The basic idea is that extra dimensions can be used to solve the hierarchy problem: the fields of the standard model must be confined to a $(3 + 1)$-dimensional subspace, embedded in a n-dimensional manifold. In the seminal works of Arkani-Hamed, Dimopoulos, Dvali and Antoniadis[3], the 4-dimensional Planck mass is related to M, the fundamental scale of the theory, by the extra-dimensions geometry. Through the Gauss law, they have found $M_{pl}^2 = M^{n+2}V_n$, where V_n is the extra dimensions volume. If V_n is large enough, M can be of the order of the weak scale. However, unless there are many extra dimensions, a new hierarchy is introduced between the compactification scale, $\mu_c = V^{-\frac{1}{n}}$, and M. An important feature of this model is that the space-time metric is factorizable, i.e., the n-dimensional space-time manifold is approximately a product of a 3-dimensional space by a compact $(n - 3)$-dimensional manifold.

Because of this new hierarchy, Randall and Sundrum[4] have proposed a higher dimensional scenario that does not require large extra dimensions, neither the supposition of a metric factorizable manifold. Working with a single S^1/Z_2 orbifold extra dimension, with three-branes of opposite tensions localized on the fixed points of the orbifold and with adequate cosmological constants as 5-dimensional sources of gravity, they have shown that the space-time metric of this model contains a redshift factor which depends exponentially on the radius r_c of the compactified dimension:

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c d\phi^2, \tag{1}$$

where k is a parameter of the order of M, x^μ are Lorentz coordinates on the surfaces of constant ϕ, and $-\pi \leq \phi \leq \pi$ with (x, ϕ) and $(x, -\phi)$ identified. The two 3-branes are localized on $\phi = \pi$ and $\phi = 0$. In fact, this scenario is well

known in the context of string theory. The non-factorizable geometry showed in Eq.(1) has important consequences. In particular, the 4-dimensional Planck mass is given in terms of the fundamental scale M by

$$M_{pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}], \tag{2}$$

in such a way that, even for large kr_c, M_{pl} is of the order of M. The other success of this scenario is that the standard model particle masses are scaled by the warp exponential factor.

C. Topological gravity: some motivations

Despite these important developments in classical Einstein gravity, background independent theories are welcome. As an example it is worth mentioning the Quantum Loop Gravity, developed mainly by Asthekar et al[5]. Also the problem of background dependence of string field theory has not been successfully addressed. The string field theory has a theoretical problem: it is only consistently quantized in particular backgrounds, which means that we have to specify a metric background in order to write down the field equations of the theory. This problem is fundamental because a unified description of all string backgrounds would make possible to answer questions about the selection of particular string vacua and in general to give us a more complete understanding of geometrical aspects of string theory[6].

Due to these developments, we regard as an important subject look for topological theories in the brane context, with the major objective to add quantum information to the Randall-Sundrum scenario. In this sense, as the first step, we study topological theories on brane-worlds in several dimensions. In this part of the work we construct topological theories in brane-worlds. The brane-world is regarded as a kink-like soliton and it appears due to a spontaneous symmetry breaking of a Peccei-Quinn-like symmetry. Topological terms are obtained by generalizing to many dimensions the axion-foton anomalous interaction.

II. TOPOLOGICAL TERMS IN BRANE-WORLDS

We implement the theory through the following action in $D = 5 + 1$:

$$S = \int d^6x \left(-\frac{1}{2\,(3!)} H_{MNP}H^{MNP} + g\varepsilon^{MNPQRS}\phi\,(z)\,H_{MNP}H_{QRS} + \frac{1}{2}\partial_M\phi\partial^M\phi + V\,(\phi) \right) \tag{3}$$

In this action, $H_{MNP} = \partial_M B_{NP} + \partial_N B_{PM} + \partial_P B_{MN}\,(M, N, ... = 0, ..., 5)$ is the field strength of an antisymmetric tensor gauge field B_{MN}. The field $B_{\mu\nu}$ has an important function in string theory: it couples correctly with the string world-sheet in a very similar way to the coupling of a gauge vector field A_M to the universe line of a point particle. The field ϕ is a real scalar field, and $V\,(\phi) = \lambda\,(1 - \cos\phi)$, is a potential that provides a phase transition.

The second term in the action (3) is a term that generalizes the coupling that appears from the anomaly of the Peccei-Quinn quasisymmetry in $D = 3 + 1$, namely, $\phi \to \phi + a$. For such, the space-time dimension is $D = 5 + 1$ and the hypersurface is a $D = 4 + 1$ world. Now we can work with the second term of (3) (considering that ϕ only depends of the z-coordinate z) in order to obtain new terms by integration by parts. Considering that the B_{MN} field weakly depends on the z-coordinate, we obtain:

$$S_{top.} = \int d^5x\, \left(k\varepsilon^{MNPQR}B_{MN}H_{PQR}\right) \tag{4}$$

This last equation shows that over the hypersurface an effective topological term appears with a coupling constant **k** that have canonical dimension of mass. The theory over the hypersurface is completely five-dimensional. This term is very similar to the Chern-Simons term, that is written in $D = 2 + 1$ with a gauge vector field A_μ. Nevertheless, the term (4) is written only with tensorial antisymmetric fields $B_{\mu\nu}$. Such term have been used to explain some peculiarities of the Cosmic Microwave Background Radiation (CMBR) within the Randall-Sundrum scenario[7]. It is interesting now to observe the properties of the action (3) in lower dimensional space-times using dimensional reduction. Thus, supposing that the fields of the action (3) are independent of the coordinate $x_M \equiv x_5$ which is not the argument of the field $\phi\,(z)$, we find a new action in $D = 4 + 1$. This action has now a vectorial gauge field reminiscent of the reduction, and contains yet the real scalar field ϕ, that again may give rise to the formation of a lower dimensional domain wall-brane. In this case, the space-time dimension is $D = 4 + 1$ and the hypersurface is a $D = 3 + 1$ universe. If we observe the theory over the solitonic hypersurface we will obtain that,

$$S_{top.} = k \int d^4x\, \left(\varepsilon^{4\nu\alpha\rho\sigma}V_{\nu\alpha}B_{\rho\sigma}\right) \tag{5}$$

If the field V_μ is identified with the potential four-vector A_μ then we obtain the action for the $B \wedge F$ model on the domain wall-brane. This action, under certain conditions, can give rise to a mechanism of topological mass generation for the field A_μ or for the field $B_{\mu\nu}$.

The discussion for lower dimensions ($D = 3 + 1$) and ($D = 2 + 1$), using the same methods, will lead us to the following topological action:

$$S_{top.} = k \int d^4x \left[\varepsilon^{\mu\nu\alpha\rho}\phi\left(z\right)\partial_\mu\phi\partial_\nu B_{\alpha\rho} + \varepsilon^{\mu\nu\alpha\rho}\phi\left(z\right)F_{\mu\nu}W_{\alpha\rho}\right] \tag{6}$$

The fields ϕ and $W_{\alpha\rho} = \partial_\alpha W_\rho - \partial_\rho W_\alpha$ emerge as degrees of freedom reminiscent of the reduction. If we work with the first term of (6) on the domain wall, we will find a different topological theory:

$$S = \int d^3x \left(g\varepsilon^{abc}\partial_a\phi B_{bc}\right) \tag{7}$$

Identifying again in (6), in the second term, the vector field W_μ as the gauge field A_μ, then we will obtain the anomalous interaction term between the real scalar field ϕ and the field A_μ . This term, rearranged on the domain wall, reduces to the Chern-Simons term.

III. TOPOLOGICAL APPROACH TO THE HIERARCHY PROBLEM

In this section we review an alternative to the central point of the Randall-Sundrum model[8], namely, the particular nonfactorizable metric. Using a topological theory, we show that the exponential factor, crucial in the Randall-Sundrum model, appears in our approach, only due to the brane existence instead of a special metric background. In order to study the hierarchy problem we choose to work with topological gravity. Motivated by current searches in the quantum gravity context, we study topological gravity of $B \wedge F$ type. Then, we can affirm that our model is purely topological because 1) the brane exists due to the topology of the parameter space of the model and 2) gravity is metric independent. We will see that these features give us interesting results when compared to the Randall-Sundrum model.

A. The Model

The model is based on the following action:

$$S = \int d^5x[\frac{1}{2}\partial_M\phi\partial^M\phi + k\varepsilon^{MNPQR}\phi H^a_{MNP}F^a_{QR} - V(\phi)]. \tag{8}$$

In this action the ϕ field is a real scalar field that is related to the domain wall. The fields H^a_{MNP} and F^a_{QR} are non-abelian gauge fields strengths and will be related to the gravitational degrees of freedom. Namely, in pure gauge theory, $H^a_{MNP} = \partial_M B^a_{NP} - \partial_N B^a_{PM} - \partial_P B^a_{MN} + gf^{abc}A^b_M B^c_{NP}$ and $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g' f^{abc}A^b_M A^c_N$. The second term of this action is a topological version of the terms studied above. The action (8) is invariant under a Peccei-Quinn symmetry transformation $\phi \to \phi + 2\pi n$. The potential is

$$V(\phi) = \lambda(1 - \cos\phi), \tag{9}$$

which preserves the Peccei-Quinn symmetry. Nevertheless, it is spontaneously broken in scales of the order of $M_{PQ} \sim 10^{10} - 10^{12}$GeV. We propose the following potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2, \tag{10}$$

which explicitly breaks the $U_{PQ}(1)$ Peccei-Quinn symmetry, in order to generate a brane in an energy close to the weak scale. With this particular choice of the potential, the existence of the brane is put on more consistent grounds. In other words, the brane appears almost exactly in an energy scale of the universe near the symmetry breaking scale of the electroweak theory. This feature was assumed in previous works without a careful justification. However, this mechanism leads to a large disparity between the Planck mass $M_{PL} \sim 10^{18}GeV$ and the scale of explicit breaking of $U_{PQ}(1)$ which is relatively close to the weak scale, $M_{ew} \sim 10^3GeV$: we assume this disparity as a new version of the hierarchy problem. Consider now the solution $\phi(x_4) = v\tanh(\sqrt{\frac{\lambda}{2}}vx_4)$. This solution defines a 3-brane embedded in

a $(4+1)$-dimensional space-time. The mass scale of this model is $m=\sqrt{\lambda}v$ and the domain wall-brane thickness is m^{-1}. With this information we can now discuss the effective theory on the domain wall-brane. An integration by parts of the topological term in the action (8) will result in

$$S \sim \int d^4x \varepsilon_{\nu\alpha\rho\lambda} B^a_{\nu\alpha} F^a_{\rho\lambda} [\lim_{r_c \to +\infty} k' \int_0^{r_c} dx_4 \partial_4 \phi(x_4)], \tag{11}$$

where r_c represents the extra dimension. This last conclusion denotes the domain wall-brane contribution to the effective four-dimensional theory. We can see that, effectively on the domain wall-brane, the theory is purely 4-dimensional (this is important) and is described by a non-abelian topological $B \wedge F$ term. It can be shown that, under parameterizations by tetrad fields, a $B \wedge F$ type action gives us

$$\int d^4x k \varepsilon^{\nu\alpha\rho\lambda} B^a_{\nu\alpha} F^a_{\rho\lambda} \to k \int d^4x \sqrt{g} R, \tag{12}$$

which is the Einstein-Hilbert action for the gravitational field, where R is the scalar curvature and g stands for the space-time metric. From Eqs. (11) and (12), we can see the relation between the Planck mass k_4 in $D=4$ and the extra dimension:

$$k_4 = \lim_{r_c \to +\infty} k' \int_0^{r_c} dx_4 \partial_4 \phi(x_4). \tag{13}$$

The limit $r_c \to +\infty$ ensures the topological stability of the domain wall-brane. By the substitution of the aforementioned solution $\phi(x_4)$ in Eq. (13), considering a finite r_c (which means that the domain wall-brane is a finite object), we can show that

$$k_4 = k' v (1 - e^{-2y})(1 + e^{-2y})^{-1}, \tag{14}$$

where $y = \sqrt{\frac{\lambda}{2}} v r_c$ is the scaled extra dimension. This result is very interesting: as our model is a topological one, the exponential factor must not appear from any special metric. Here, the exponential factor appears only due to the domain wall-brane existence. As in the Randall-Sundrum model, even for the large limit $r_c \to +\infty$, the 4-dimensional Planck mass has a specific value. This is the reason why we believe that our approach can be useful to treat the hierarchy problem. It is possible to obtain scaled masses to the confined matter using zero modes attached to the domain wall-brane. We address the fact that we only use the domain wall-brane characteristics.

IV. ON AXION PHYSICS: SOME PERSPECTIVES

In this section we make a brief review of recent developments about axion physics and propose some new perspectives in this area. The motivations for these discussions are only mathematical: the topological terms studied in the sections above can give us some insights. Extra dimension physics gives new viewpoints for axion studies. In particular, an interesting problem is how to generate the axion scale ($f_a = 10^{10} \sim 10^{12}$ Gev) that is intermediate to the Planck scale ($M_p \approx 10^{18}$ Gev) and to the electroweak symmetry breaking scale ($M_{ew} \approx 1$ Gev). Such problem may be solved naturally by a $5D$ orbifold field theories[9] if the axion originates from a higher-dimensional parity-odd field C_M:

$$S_{5D} = \int d^4x dy \sqrt{-G} (\frac{1}{l^2} C^2_{MN} + \frac{\kappa}{\sqrt{-G}} \epsilon^{MNPQR} C_M F^a_{NP} F^a_{QR} + \ldots) \tag{15}$$

The action contains a Chern-Simons coupling to the $U(1)$ gauge field C_M; F^a_{NP} is a standard model non-abelian field strength. The axion appears as the C_5 field, a component of C_M, together with the correct scale f_a.

On the other hand, interesting studies have been made in gauge fields localization procedures. For the brane of the δ-function type, Dvali et al.[10] have shown that, for gauge fields, localization holds for specific distances but there is an effect of dissipation of cosmic radiation to extra dimensions for large distances. The lagrangian is the following:

$$L = -\frac{1}{4g^2} F^2_{AB} - \frac{1}{4e^2} F^2_{\mu\nu} \delta(y) + \ldots \tag{16}$$

The first term refers to the bulk physics while the second one, to the brane physics ($A, B = 1, \ldots, 5$ and $\mu, \nu = 1, \ldots, 4$). On the brane, the F^2 term can be generated by radiative corrections due to localized matter fields. An interesting problem is to analyze axion physics in this context in order to see if the same phenomenon happens for axions. In

$D = 4$, the axion field may be described by an antisymmetric field $B_{\mu\nu}$. A nice way to study this question would be to start with the following model in $D = 5$:

$$L = H^2_{ABC} + H^2_{\mu\nu\alpha}\delta(y) + \dots \tag{17}$$

In another way (equivalent theories on branes) we can try in $D = 5$ the following model:

$$L = H^2_{ABC} - F^2_{AB} + \delta(y)[\varepsilon^{\mu\nu\alpha\beta}B_{\mu\nu}F_{\alpha\beta} + A_\mu A^\mu] \tag{18}$$

On the brane we have a partially topological theory that is equivalent to a H^2 theory. In this case we have to consider that we can get the equations of motion of the brane physics independently of the bulk physics. We can do this if we consider low energies in the $D = 4$ world. In this case we can construct a mechanism of axion localization on branes in a different fashion.

V. CONCLUSIONS

By a procedure of dimensional reduction, we have constructed several Chern-Simons-like topological terms, in abelian and in non-abelian theories. The domain wall-brane has been simulated by a kink-like soliton embedded in a higher dimensional space-time and it has emerged due to a spontaneous symmetry breaking of a specific discrete symmetry, namely, a Peccei-Quinn-like symmetry. We have shown that a simple topological model in field theory has the necessary features to solve the Gauge Hierarchy problem in a very similar way to the one found by Randall and Sundrum. With this model we have built a stable 3-brane (a domain wall-brane) that simulates our four-dimensional Universe and we have argued the possibility of topological gravity localization. Because of these facts, the exponential factor appears only due to the existence of the domain wall-brane and not from a special metric. We have discussed some of our perspectives on axion physics. We have noted that we can construct a mechanism of axion localization using a partially topological field theory. Studies related to generation of axion scales will follow in a forthcoming paper.

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